REMOTE AND DISTRIBUTED QUANTUM HYPOTHESIS TESTING

Auteurs

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Partenaires

LTCI

QUANTUM HYPOTHESIS TESTING

- Goal: decide between two alternative explanations for the data observed.
- Two hypotheses:

$$\begin{cases} \mathcal{H}_0 : \rho^{\otimes n} & \text{(null hypothesis)} \\ \mathcal{H}_1 : \sigma^{\otimes n} & \text{(alternative hypothesis)} \end{cases}$$

- Decision: a POVM {Π0, Π1}.
- Two types of decision error:

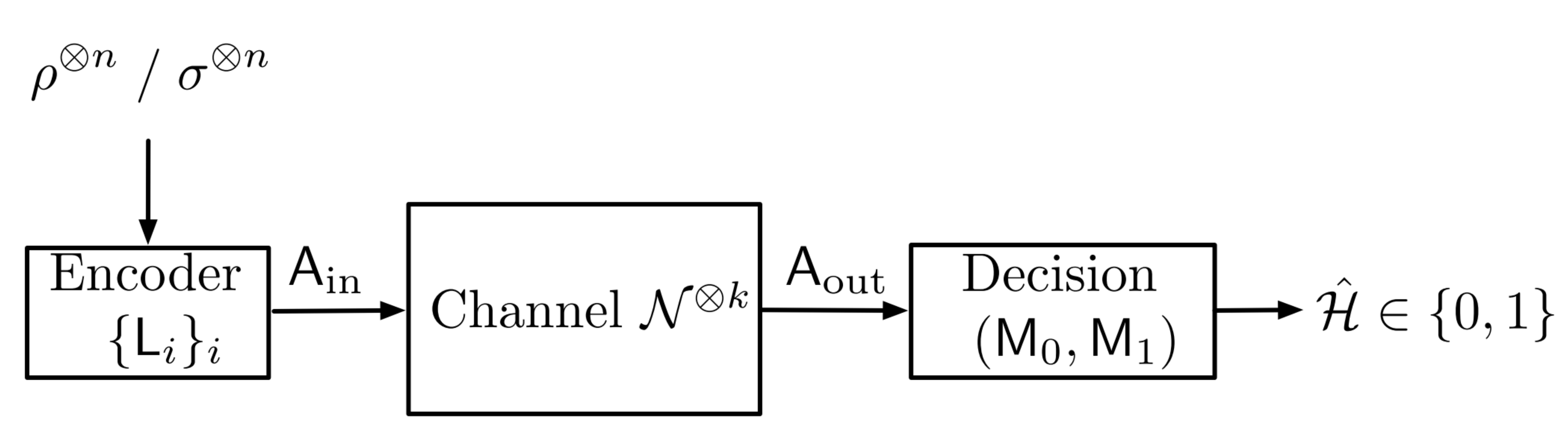
$$\alpha_n = \mathrm{Tr}(\Pi_1 \rho^{\otimes n})$$
$$\beta_n = \mathrm{Tr}(\Pi_0 \sigma^{\otimes n})$$

- Objective: minimize the Type II error under a constraint on the Type I error.
- Best exponential decay rate of the Type II error:

$$\lim_{n\to\infty} -\frac{1}{n} \log \beta_n^\star = D(\rho\|\sigma)$$

PROBLEM DEFINITION

- An encoder receives a quantum state.
- It applies a quantum operation.
- It sends the result over the channel.
- The channel can be used *k* times.
- A decision center recevies the resulting quantum state.
- It applies a measurement and takes a guess.



RESULTS

- We restrict our attention to a subset of channels.

$$\mathcal{N}_1 := \{\mathcal{N} \mid \exists \varrho \succeq 0 \text{ satisfying trace}(\varrho) = 1, \ \mathrm{rank}(\mathcal{N}(\varrho)) = N_{\mathrm{out}}\}$$

- We then divide this set into two classes:

$$\mathcal{C} := \{\mathcal{N} \in \mathcal{N}_1 \mid \forall \tau \succeq 0 \text{ with trace}(\tau) = 1 \text{ it holds that } \mathrm{rank}(\tau) = N_{\mathrm{out}}\} \qquad \bar{\mathcal{C}} := \mathcal{N}_1 \setminus \mathcal{C}.$$

- We get that the best exponential rate of decay θ for the type II error under a constraint epsilon on the Type I error is:

Theorem 1: If $\mathcal{N} \in \mathcal{C}$, then for all $\epsilon \in [0, 1)$:

$$\theta^*(\epsilon) = 0.$$

and if $\mathcal{N} \in \bar{\mathcal{C}}$, then for all $\epsilon \in [0, 1)$:

$$\theta^*(\epsilon) = D(\rho\|\sigma).$$

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